

ever for an umklapp (U.K.) process in which $K \neq 0$, \mathbf{q} depends not only on θ , but also on the particular reciprocal lattice vector \mathbf{K} used. The latter depends on the initial state \mathbf{k} .

Figure 10 shows a cross section of the Brillouin zone for a bcc lattice, taken in a [001] plane. The circles are cross sections of the Fermi surface and the dashed square is a zone for phonons, centered on the state \mathbf{k} . The U.K. processes are those for which the final state \mathbf{k}' lies on that portion of the circle centered at 0 which is outside the dashed square. The dependence of $|\mathbf{q}|$ on the initial state \mathbf{k} can be most easily seen by taking $\theta = 180^\circ$ and \mathbf{k} first in the [100] and then in the [110] direction. For the latter direction $|\mathbf{q}|$ is about $\frac{1}{2}$ as large as for \mathbf{k} in the [100].

Normal processes must use longitudinal phonons, at least for those directions in which a separation into longitudinal and transverse modes is possible, because the term $\hat{\epsilon}_{\mathbf{q},\mathbf{p}} \cdot \mathbf{s}$ becomes $\hat{\epsilon}_{\mathbf{q},\mathbf{p}} \cdot \mathbf{q}$ and this is zero for a transverse mode. For a U.K. process \mathbf{s} is not parallel to \mathbf{q} and transverse phonons participate; indeed the small sound velocity for transverse phonons emphasizes those U.K. processes which use transverse phonons.

We should like to obtain the anisotropy of $\tau(\mathbf{k})$. To do this accurately one would need to choose a direction \mathbf{k} , compute \mathbf{q} , $c_{\mathbf{q},\mathbf{p}}$, and $\hat{\epsilon}_{\mathbf{q},\mathbf{p}}$ for a large number of points \mathbf{k}' on a Fermi sphere and evaluate Eq. (29). This is a major computational task; a simpler but considerably less accurate procedure is to consider only scattering in two dimensions and evaluate the θ integral in Eq. (29) for fixed ϕ . Some of the loss of accuracy comes from the fact that for some \mathbf{k} directions the θ integral depends strongly on the value of ϕ chosen. For example, if one chose \mathbf{k} in the 110 direction and replaced the Brillouin zone for phonons by a sphere of equal volume, there is ϕ symmetry in the sense that $|\mathbf{q}|$ for fixed θ is independent of ϕ , although $c_{\mathbf{q},\mathbf{p}}$ and $\hat{\epsilon}_{\mathbf{q},\mathbf{p}} \cdot \mathbf{s}$ are not; on the other hand for \mathbf{k} in a 111 direction this is not true.

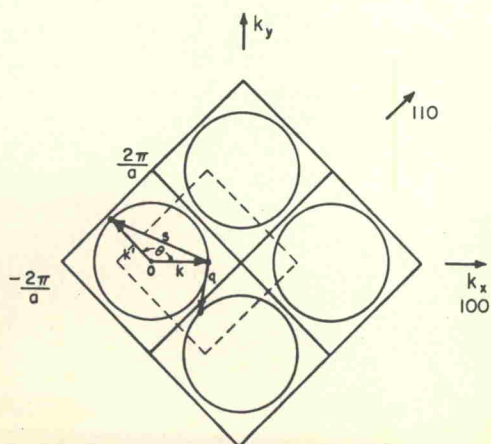


FIG. 10. Cross section, in [001] plane, of Brillouin zones for bcc metal.

We estimated the anisotropy in $\tau(\mathbf{k})$ by evaluating

$$I(\mathbf{k}) = \int_0^{2\pi} \frac{(1 - \cos\theta) |\sin\theta| [\{JS\}(\theta)]^2 d\theta}{|\mathbf{q}|^2} \quad (31)$$

for \mathbf{k} in the [100], [110], and [111] directions. This is a very crude procedure, which not only replaces the three dimensional integral in Eq. (29) by a two dimensional one, but also considers the velocity of sound, the polarization factor $\hat{\epsilon}_{\mathbf{q},\mathbf{p}} \cdot \mathbf{s}$ and the density of states as constants. The calculation will underestimate the anisotropy in τ since it ignores the anisotropy of the velocity of sound (Table V).

We obtained the values of $(JS)^2$ from Bailyn²⁰; in Table VI we list his values of $u^3(JS)^2$ vs $u = \sin\theta/2$ for potassium and lithium. Potassium was chosen as representative of all the alkalis except lithium, which differs from the others in that $(JS)^2$ goes through a zero near

TABLE VI. Scattering functions for K and Li using Bailyn's values of $u^3(JS)^2$.

$u = \sin\theta/2$	θ degrees	$(1 - \cos\theta) \times \sin\theta$	$u^3(JS)^2$ for K	$u^3(JS)^2$ for Li	$F(\theta)$ for K	$F(\theta)$ for Li
0.00	0	0	0	0	0.00	0.00
0.10	11.5	0.004	0	0	0.00	0.00
0.20	23	0.03	0.005	0.005	0.02	0.02
0.30	35	0.10	0.035	0.035	0.13	0.13
0.40	47	0.23	0.090	0.090	0.32	0.32
0.50	60	0.43	0.190	0.190	0.65	0.65
0.60	74	0.70	0.315	0.265	1.02	0.86
0.65	81	0.83	0.345	0.275	1.04	0.83
0.70	89	0.98	0.375	0.265	1.07	0.76
0.75	97	1.11	0.393	0.220	1.03	0.58
0.80	106	1.22	0.400	0.125	0.95	0.30
0.85	116	1.30	0.385	0.025	0.82	0.05
0.90	128	1.28	0.340	0.010	0.60	0.02
0.95	144	1.07	0.305	0.025	0.38	0.03
0.97	152	0.88	0.300	0.033	0.29	0.03
0.98	157	0.70	0.296	0.035	0.23	0.03
0.99	164	0.54	0.293	0.040	0.17	0.02
1.80	180	0.00	0.290	0.042	0.00	0.00

$\theta = 120^\circ$. We also tabulate

$$F(\theta) = (JS)^2 (1 - \cos\theta) \sin\theta, \quad (32)$$

for potassium and lithium. Values of \mathbf{q} as a function of θ for \mathbf{k} in the [100], [110], and [111] directions were obtained by measuring on a diagram such as that in Fig. 10, the integral of Eq. (31) plotted as a function of θ , and $I(\mathbf{k})$ evaluated graphically. The results are shown in Table VII; we have also evaluated $I(\mathbf{k})$ counting U.K. processes only. This takes account of the large velocity of sound for the longitudinal phonons by not counting these processes at all.

As pointed out before the integration for $\mathbf{k}(111)$ is unrepresentative since the possible scattering processes depend strongly on the particular great circle on the Fermi sphere for which we have chosen to do the θ integration; because of the strong ϕ dependence no two dimensional integration will give a very meaningful